(6.7) 1/2/11

A bus stopped to pick up riders. A woman named Charis is running at a constant velocity of 5 m/sec. When she is 11 m behind the front door the bus pulls away with a constant acceleration of 1 m/sec². How long will it take the woman to reach the front door of the bus?

$$t = 5-\sqrt{3}$$

$$x = 5(5-\sqrt{3}) - 11$$

$$= 14 - 5\sqrt{3}$$

$$= \frac{(5-\sqrt{3})^2}{2}$$

$$= \frac{25+3-10\sqrt{3}}{2}$$

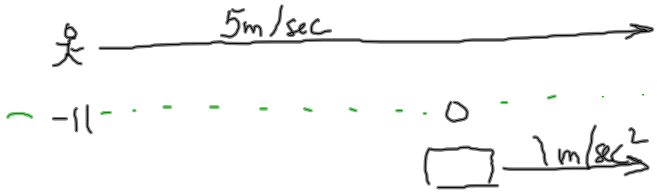
$$14 - 5\sqrt{3}$$

$$\frac{t^2}{2} = 5t - 11$$

$$t^2 - 10t + 22 = 0$$

$$t = \frac{10 \pm \sqrt{100-88}}{2}$$

$$= 5 \pm \sqrt{3}$$



acceleration

woman: $0 \, m/sec^2$

bus : $1 \, m/sec^2$

velocity

woman: $\int 0 \, dt = c$

$5 \, m/sec$

bus: $\int 1 \, dt = t + C$

velocity of the bus at time $t=0$ $= 0$

$t + C = 0$ (when $t=0$)

$0 + C = 0$

$\Rightarrow C = 0$

$V_w(t) = 5$

$V_b(t) = t$

position:

woman:

$x_w(t) = \int v(t)\,dt = \int 5 \, dt$

$= 5t + C$ @ $t=0, x=-11$

$= 5t - 11$

bus: $X_b(t) = \int t \, dt = \frac{t^2}{2} + C$

at $t=0, x=0$

$X_b(t) = \frac{t^2}{2}$

Suppose a particle moves on the coordinate line so that velocity $v(t) = t^2 - 2t$ m/sec.

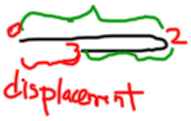
* Find the displacement of the particle during the time interval $0 \le t \le 3$
* Find the distance travelled by the particle during the time interval $0 \le t \le 3$

displacement: $x(3) - x(0)$

$x(t)$ is an antiderivative of $v(t)$

FTC: displacement $= \int_0^3 v(t)\,dt$

distance: $\rightarrow$ find where particle chgs direction:



$v(t) = 0$ ... $t^2 - 2t = t(t-2) = 0$

zero velocity @ $t = 0$, $t = 2$

sign chart of velocity says particle chgs direction at $t = 2$

$\rightarrow$ distance $= |\text{displacement}(0 \to 2)| + |\text{displacement}(2\text{-}3)|$

distance $= \int_0^3 |v(t)|\,dt$

(6.7) 1/7/11

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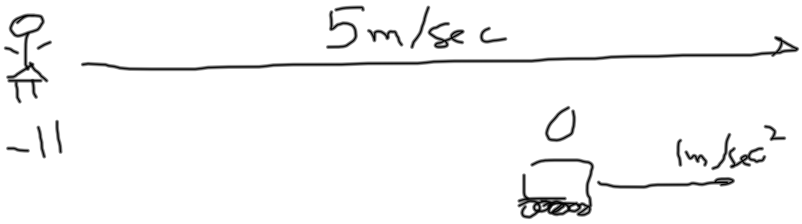
$$@\ x = 5(5-\sqrt{3})-11 = 14-5\sqrt{3}$$

$$@\ x = \frac{(5-\sqrt{3})^2}{2} = \frac{(25+3-10\sqrt{3})}{2}$$



acceleration

woman: $0 \, m/sec^2$

bus : $1 \, m/sec^2$

$a_w(t) = 0$

$a_b(t) = 1$

velocity

woman $v_w(t) = \int 0 \, dt = C$

at $t=0$, $v=5$ $\therefore C=5$

$v_b(t) = \int 1 \, dt = t + C$

at $t=0$, $v=0$ $\quad 0+C=0 \therefore C=0$

$V_w(t) = 5$

$V_b(t) = t$

position

Woman: $x(t) = \int v(t) \, dt = \int 5 \, dt = 5t + c$

at $t=0$, $x=-11$ $\quad 5(0)+C=-11$

$x_w(t) = 5t - 11$

bus $x_b(t) = \int v(t) \, dt = \int t \, dt = \frac{t^2}{2} + C$

at $t=0$, $x_b = 0$ $\therefore C=1$

$x_w(t) = 5t - 11 = X_b(t) = \frac{t^2}{2}$

$\frac{t^2}{2} = 5t - 11 \Rightarrow t^2 - 10t + 22 = 0$

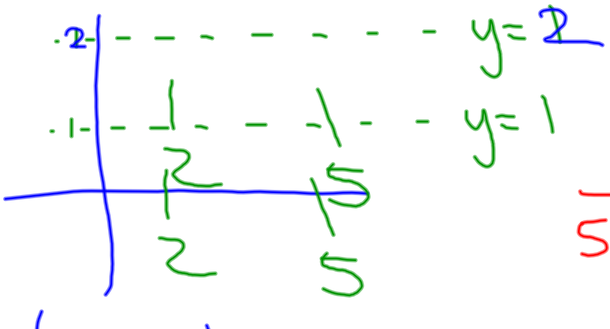
$t = \frac{10 \pm \sqrt{100-88}}{2} = 5 \pm \sqrt{3}$

Definite Integral $\equiv$ Accumulator of instantaneous rates of chg

Definite Integral $\equiv$ Infinite sum of function values

"Avg value" of $f(x)$ over $[a,b]$ $= \dfrac{1}{(b-a)} \displaystyle\int_a^b f(x)\,dx$

$$\frac{1}{3}\int_2^5 2\,dx$$



$$\frac{1}{5-2}\int_2^5 1\,dx$$

$$\frac{1}{3}(2x)\Big|_2^5 = \frac{1}{3}(10-4) = \frac{1}{3}(6) = 2$$

$$\frac{1}{3}(x)\Big|_2^5 = \frac{1}{3}[5-2] = \frac{1}{3}(3) = 1$$

what is the average rate of chg of $f(x) = x^2+1$ between $x=-1$ & $x=3$

$f(-1) = (-1)^2+1 = 2$

$f(3) = (3)^2+1 = 10$

$(-1,2) \longrightarrow (3,10)$

$\text{avg roc} = \frac{10-2}{3-(-1)} = \frac{8}{4} = 2$

avg rate of chg = avg value of the rate of chg function

= avg value of derivative

$f(x) = x^2+1$

$f'(x) = 2x$

$\text{avg value of rate of change} = \frac{1}{3-(-1)} \int_{-1}^{3} (2x)\,dx$

$= \frac{1}{4}(x^2)\Big|_{-1}^{3}$

$= \frac{1}{4}(9-(-1)^2)$

$= \frac{1}{4}(8) = 2$

def^n

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x \to 0} \underbrace{\sum_{k=1}^{n} f(x_k^*)\,\Delta x_k}_{\text{Riemann sum}}$$

"practical aspect"

$$\int_a^b f(x)\,dx \equiv \text{area between } f(x) \text{ \& } x\text{-axis}$$

[[assuming $f(x) \geq 0$]]

evaluation

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

if $F(x) = \int f(x)\,dx$